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Heterotic Hyper-Kähler flux backgrounds

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ABSTRACT: We study Heterotic supergravity on Hyper-Kähler manifolds in the presence of non-trivial warping and three form flux with Abelian bundles in the large charge limit. We find exact, regular solutions for multi-centered Gibbons-Hawking spaces and Atiyah-Hitchin manifolds. In the case of Atiyah-Hitchin, regularity requires that the circle at infinity is of the same order as the instanton number, which is taken to be large. Alternatively there may be a non-trivial density of smeared five branes at the bolt.

KEYWORDS: Superstring Vacua, Superstrings and Heterotic Strings, Flux compactifications, Black Holes in String Theory

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1 Introduction

Heterotic flux backgrounds are interesting models of string backgrounds. There is no in principle impediment from using the R-NS string worldsheet formalism although compact models with minimal or no supersymmetry remain challenging to construct.

In this work we study local Heterotic flux backgrounds on Hyper-Kähler four manifolds: in particular the Gibbons-Hawking spaces [1] and the Atiyah-Hitchin manifold [2]. Key to our configurations is that the gauge fields are Abelian and we take the large charge limit such that $\text{Tr}F \wedge F$ dominates $\text{Tr}R \wedge R$ in the Bianchi identity. This large-charge limit has been previously studied on the Eguchi-Hanson space [3] and the conifold [4, 5] and this type of limit is familiar from the large-charge supergravity limit crucial to the development of holography [6] in type II and M-theory.

Our strategy is to first compute explicit solutions to the Hermitian-Yang-Mills equations on Hyper-Kähler spaces and then backreact them on the geometry. This backreaction affects only the conformal mode of the metric but generates a non-trivial three-form flux.

According to Aspinwall [7] we are studying *Goldilocks* theories with just the right amount of supersymmetry; perhaps then not surprisingly we solve the supergravity background exactly. For the Atiyah-Hitchin background it is nonetheless somewhat impressive that we can both solve Hermitian-Yang-Mills exactly and integrate analytically the resulting Poisson equation for the backreaction of this instanton.

The instanton we use on the Atiyah-Hitchin manifold is well known from a classic duality paper by Ashoke Sen [8]. Perhaps the central result of our work is that the Heterotic backreaction of this instanton can be made regular. This is somewhat non-trivial since the negative mass of Atiyah-Hitchin induces a negative warp factor thus violating the desired signature of space-time. We circumvent this in two ways: first by allowing the asymptotic circle to be large and secondly by including smeared five-brane sources.

We also study the presence of electric H -flux and fundamental strings. The electric flux modifies the BPS equations in a straightforward way and for each solution with magnetic H -flux, the electric flux can be added through a harmonic function on the Hyper-Kähler manifold. We analyze limits in which we recover AdS_3 geometries but these reduce to the known $\text{AdS}_3 \times S^3 \times \text{HK}_4$.

Upon completing this work we were made aware that our BPS equations have turned up in five dimensional supergravity. The local equations we study can essentially be found in [9, 10] but with very different global and regularity requirements. This is not surprising since we can dimensionally reduce our solutions on \mathbb{R}^5 to get solutions of ungauged five dimensional supergravity. In addition, these equations also turn up in type II supergravity for T^2 fibrations over Hyper-Kähler spaces and the type II analogue of the Gibbons-Hawking solutions we find have been analyzed in [11]. It is straightforward to convert our solutions on the Atiyah-Hitchin manifold to such type II backgrounds.

2 Hyper-Kähler heterotic backgrounds

The primary backgrounds we consider are of the form $\mathbb{R}^{1,5} \times \text{HK}_4$ where HK_4 is a warped Hyper-Kähler-four manifold. We consider a non-trivial three-form flux $\mathcal{H}_{(3)}$, dilaton Φ and Heterotic gauge field F . The background metric ansatz is:

$$ds_{10}^2 = ds_{1,5}^2 + H ds_4^2 \tag{2.1}$$

where ds_4^2 is an Hyper-Kähler metric on a four-manifold HK_4 and H a conformal factor. The BPS equations are fairly standard:

$$e^{2\phi} = H \tag{2.2a}$$

$$\mathcal{H}_{(3)} = - *_4 dH \tag{2.2b}$$

$$J_a \lrcorner F = 0, \quad a = 1, 2, 3 \tag{2.2c}$$

where the $*_4$ is the Hodge dual w.r.t. the Hyper-Kähler metric on HK_4 and J_a are the three Kähler forms.

A major difficulty in finding explicit solutions of Heterotic supergravity with non-trivial three-form flux is to satisfy the Bianchi identity at the appropriate order in α' . Following

earlier works by some of the authors [3–5], our strategy will be to work in a large (fivebrane) charge limit, ensuring that the contribution of the $\text{Tr}R \wedge R$ term is subdominant and can be consistently neglected. The Heterotic Bianchi identity simplifies to

$$d\mathcal{H}_{(3)} = \alpha' \text{Tr} F \wedge F \tag{2.3}$$

implying from the three-form ansatz (2.2b) that:

$$d *_4 dH = -\alpha' \text{Tr} F \wedge F. \tag{2.4}$$

2.1 Principal torus bundles and type IIA/IIB solutions

We consider a more general class of backgrounds, that can be viewed as local models of the principal torus bundles over wrapped K3 surfaces introduced in [12] and discussed in many works including [13–16]. They generalize the solutions based on Eguchi-Hanson space presented in [17]. The general ansatz for such principal two-torus bundle $T^2 \hookrightarrow M_6 \xrightarrow{\pi} HK_4$ over a Hyper-Kähler four-manifold is of the form

$$ds_{10}^2 = ds_{1,3}^2 + H ds_4^2 + \frac{U_2}{T_2} |dx + Tdy + \pi^* \alpha|^2, \tag{2.5}$$

where α is a connection one-form on HK_4 such that $\vartheta = dx + Tdy + \pi^* \alpha$ is a globally defined one-form on M_6 with

$$\frac{1}{2\pi} d\vartheta = \pi^* \varpi, \quad \varpi = \varpi_1 + T\varpi_2, \quad \varpi_i \in H^2(HK_4, \mathbb{Z}), \tag{2.6}$$

and by supersymmetry

$$J^a \wedge \varpi = 0, \quad a = 1, 2, 3. \tag{2.7}$$

The expression for the three-form becomes then

$$\mathcal{H}_{(3)} = - *_4 dH - \frac{\alpha' U_2}{T_2} \text{Re} (*_4 d\vartheta \wedge \bar{\vartheta}). \tag{2.8}$$

By an appropriate choice of $\varpi \in H^2(HK_4, \mathbb{Z})$ one can find solutions with $d\mathcal{H}_{(3)} = 0$, which can also be obtained as supersymmetric solutions of type IIA or type IIB supergravity with NS-NS fluxes, as was discussed in [3] and [18].

3 Gibbons-Hawking: ALE and ALF

We can solve explicitly (2.2) and (2.4) for the multicentered Gibbons-Hawking ALE and ALF spaces, that we denote collectively by M_{GH} . The corresponding Hyper-Kähler metrics are given by:¹

$$ds_4^2 = V(x)^{-1} (d\tau + \omega)^2 + V(x) dx \cdot dx, \tag{3.1a}$$

$$dV = *_3 d\omega, \tag{3.1b}$$

$$V = \epsilon + 2m \sum_{i=1}^k \frac{1}{|\mathbf{x} - \mathbf{x}_i|}, \tag{3.1c}$$

¹As usual $*_3$ is the Hodge dual on \mathbb{R}^3 .

where $\epsilon = 0$ gives the ALE (multi Eguchi-Hanson) series and $\epsilon = 1$ the ALF (multi Taub-NUT) series. The periodicity of τ is determined by expanding around a pole of $V(\mathbf{x})$ to be:

$$\tau \sim \tau + 8\pi m, \tag{3.2}$$

and the triplet of Kähler forms is given by

$$J_a = (d\tau + \omega) \wedge dx^a - V *_3 dx^a, \quad a = 1, 2, 3. \tag{3.3}$$

We will consider heterotic supergravity solutions for warped ALE or ALF spaces supported by Abelian gauge bundles. To explicitly write the gauge fields we denote

$$V_i = \frac{2m}{|\mathbf{x} - \mathbf{x}_i|}, \quad d\omega_i = *_3 dV_i; \tag{3.4}$$

Then a representative of the topologically non-trivial gauge fields is locally given by

$$A_i = \omega_i - \frac{V_i}{V} (d\tau + \omega). \tag{3.5}$$

We note that

$$\sum_{i=1}^k A_i = \epsilon \frac{d\tau + \omega}{V} - d\tau \tag{3.6}$$

is topologically trivial since $V^{-1}(d\tau + \omega)$ is globally defined. Thus there are $(k - 1)$ non-trivial gauge fields, in agreement with the $(k - 1)$ non-trivial two-cycles. The corresponding field strengths are

$$F_i = dA_i = V *_3 d \left[\frac{V_i}{V} \right] - d \left[\frac{V_i}{V} \right] \wedge (d\tau + \omega). \tag{3.7}$$

It is straightforward to see that F_i are anti-self dual and solve Hermitian Yang-Mills²

$$*_4 F_i = -F_i, \quad J_a \wedge F_j = 0. \tag{3.8}$$

For the Bianchi identity (2.4) we compute

$$F_i \wedge F_j = -2V *_3 d \left[\frac{V_i}{V} \right] \wedge d \left[\frac{V_j}{V} \right] \wedge (d\tau + \omega) \tag{3.9}$$

$$d *_4 d \left[\frac{V_i V_j}{V} \right] = -F_i \wedge F_j \tag{3.10}$$

so that if we take

$$F = \frac{1}{4m} \sum_{i=1}^k dA_i \mathbf{q}_i \cdot \mathcal{T}, \tag{3.11}$$

²For a one form α on \mathbb{R}^3 we have $*_4 \alpha = -(d\tau + \omega) \wedge *_3 \alpha$. In particular this means that a function which is invariant under the $U(1)$ generated by ∂_τ is harmonic on the Gibbons-Hawking space iff it is harmonic on \mathbb{R}^3 .

where $\mathcal{T} \in U(1)^{16}$ is in the Cartan subalgebra of $E_8 \times E_8$ or $SO(32)$ and \mathbf{q}_i the corresponding charge vectors, and solve (2.4), we get the general solution:³

$$H = \delta + h(\mathbf{x}) + \frac{\alpha'}{8m^2V} \sum_{i,j=1}^k V_i V_j \mathbf{q}_i \cdot \mathbf{q}_j. \tag{3.12}$$

Here $\delta = 0, 1$ is an integration constant (not to be confused with ϵ a similar integration constant in the Gibbons-Hawking warp factor V) and $h(\mathbf{x})$ is any harmonic function on M_{GH} . Taking $h(\mathbf{x})$ to be invariant under ∂_τ we have

$$h(\mathbf{x}) = \frac{1}{m} \sum_{\alpha} \frac{q_{\alpha}}{|\mathbf{x} - \mathbf{x}_{\alpha}|} \tag{3.13}$$

corresponding to mobile neutral five-brane sources inserted at \mathbf{x}_{α} .

In appendix A we show how the two center solution is related to the Eguchi-Hanson solution that was discussed in particular in [3].

3.1 Five-brane and magnetic charges

At infinity we can compute the five-brane charge using (2.2b) and (3.12). We have

$$\mathcal{H}_{(3)} = (d\tau + \omega) \wedge *_3 dH \tag{3.14}$$

and so⁴

$$dH = -\frac{\alpha'}{4mkr^2} \left(\sum_{i,j=1}^k \mathbf{q}_i \cdot \mathbf{q}_j \right) dr - \frac{1}{m} \sum_{\alpha} \frac{q_{\alpha}}{r^2} + \dots \tag{3.15a}$$

$$\mathcal{H}_{(3)} = \left[-\frac{\alpha'}{4mk} \sum_{i,j=1}^k \mathbf{q}_i \cdot \mathbf{q}_j - \frac{1}{m} \sum_{\alpha} q_{\alpha} \right] (d\tau + \omega) \wedge \Omega_2 + \dots \tag{3.15b}$$

and the Maxwell five-brane charge is

$$\mathcal{Q}_M = \frac{1}{4\pi^2 \alpha'} \int_{S^3/\mathbb{Z}_k} \mathcal{H}_{(3)} = -\frac{2}{k^2} \sum_{i,j=1}^k \mathbf{q}_i \cdot \mathbf{q}_j - \sum_{\alpha} \frac{8q_{\alpha}}{k}. \tag{3.16}$$

One can also define a Page charge, which is quantized, as:

$$\begin{aligned} \mathcal{Q}_P &= \frac{1}{4\pi^2 \alpha'} \int_{S^3/\mathbb{Z}_k} (\mathcal{H}_{(3)} - A \wedge F) \\ &= -\sum_{\alpha} \frac{8q_{\alpha}}{k} \in \mathbb{Z}. \end{aligned} \tag{3.17}$$

³we have chosen to work with Hermitian gauge fields, normalized as $\text{Tr } \mathcal{T}_{\alpha} \mathcal{T}_{\beta} = 2\delta_{\alpha\beta}$.

⁴The volume form of a three-sphere is

$$\begin{aligned} ds_{S^3}^2 &= \frac{1}{4} \left[\frac{1}{4m^2} (d\tau + \omega)^2 + d\Omega_2^2 \right] \\ 2\pi^2 &= \int \text{vol}(S^3) = \frac{1}{8} \int \frac{1}{2m} (d\tau + \omega) \wedge \Omega_2. \end{aligned}$$

Magnetic charges. We take a basis of two cycles to be Δ_i where the poles of the Δ_i are at x_i and x_{i+1} . Then the matrix of magnetic charges associated with the Abelian gauge bundle are:

$$\begin{aligned}
 q_{i,j} &= \frac{1}{2\pi} \int_{\Delta_j} F_i = \frac{\mathbf{q}_i \cdot \mathcal{T}}{8\pi m} \left(\int d\tau \right) \int_{x_j}^{x_{j+1}} d \left[\frac{V_i}{V} \right] \\
 &= \mathbf{q}_i \cdot \mathcal{T} [\delta_{j+1,i} - \delta_{j,i}] .
 \end{aligned}
 \tag{3.18}$$

3.2 Partial blow-down limits and fivebranes

The function $V(\mathbf{x})$ has k -poles, now suppose that k' of these poles are co-incident, which correspond to a partial blow-down limit of the ALE or ALF space. In the present situation some of the Abelian instantons (3.7) become point-like as the corresponding two-cycles shrink and we expect heterotic five-brane to appear. We now check that in the region around such a pole we obtain the near horizon of five-brane solution of Callan, Harvey and Strominger [19, 20] where the three-sphere is orbifolded by $\mathbb{Z}_{k'}$.

For simplicity we set $\mathbf{x}_j = 0$ for $j = 1, \dots, k'$, and in the neighborhood of this pole the functions H and V behaves like

$$H \stackrel{r \rightarrow 0^+}{\simeq} \frac{1}{r^2} \frac{\alpha'}{2k'm} Q_5 \quad , \quad V \stackrel{r \rightarrow 0^+}{\simeq} \frac{4mk'}{r^2}
 \tag{3.19}$$

hence the solution approaches

$$ds_{10}^2 \stackrel{r \rightarrow 0^+}{\simeq} ds_{1,5}^2 + 2\alpha' Q_5 \left[\frac{dr^2}{r^2} + \frac{r^2}{4} \left(\sigma_1^2 + \sigma_3^2 + \left(\frac{\sigma_3}{2k'm} \right)^2 \right) \right]
 \tag{3.20a}$$

$$\mathcal{H}_{(3)} \stackrel{r \rightarrow 0^+}{\simeq} \frac{\alpha' Q_5}{2} \sigma_1 \wedge \sigma_2 \wedge \frac{\sigma_3}{2k'm}
 \tag{3.20b}$$

where the five-brane charge is given by:

$$Q_5 = \sum_{i,j=1}^{k'} \mathbf{q}_i \cdot \mathbf{q}_j .
 \tag{3.21}$$

3.3 Double scaling limit

For the two-center Eguchi-Hanson solution ($k = 2$) there exists an interesting double scaling limit [3], defined as:

$$g_s \rightarrow 0, \quad \lambda := \frac{g_s \sqrt{\alpha'}}{a} \text{ fixed and finite} ,
 \tag{3.22}$$

where a is the distance between the two centers. This limit decouples the asymptotically locally Euclidian region, and λ becomes the effective coupling constant of the interacting string theory.

In the spherical coordinates reviewed in appendix A the metric of the solution becomes

$$ds^2 = ds_{1,5} + \frac{\alpha' Q_5}{2} \left[\frac{dr^2}{r^2(1 - \frac{a^4}{r^2})} + \frac{1}{4} \left(1 - \frac{a^4}{r^2} \right) \sigma_3^2 + d\Omega_2^2 \right]
 \tag{3.23}$$

and the corresponding heterotic string theory admits an exactly solvable worldsheet CFT. This space has an asymptotic linear dilaton hence admits a holographic description as a little string theory [21] but, unlike the CHS background, is given by a smooth solution of heterotic supergravity.

A double scaling limit can be described in principle for arbitrary k . Let us define $\mathbf{x}_i = a\mathbf{y}_i$ where the coordinates \mathbf{y}_i are dimension-less and a is a common scale factor. The double-scaling limit can be then described exactly as before by eq. (3.22); in practice the double scaling limit amounts to setting $\delta \rightarrow 0$ in (3.12).

It would be interesting to check if one could derive a worldsheet CFT for the double scaled solutions when $k > 2$, in particular whenever the centers are arranged following a simple pattern, for instance a homogeneous distribution on a circle.

4 Atiyah-Hitchin

The Atiyah-Hitchin space M_{AH} is a four-dimensional smooth manifold with an explicit Hyper-Kähler metric which at long distances approximates Taub-NUT with a negative mass parameter. The original work is [2, 22] and an interesting simplification was given in [23]. Our notation will follow a more recent work [10] where M_{AH} was used as a potential Euclidean Hyper-Kähler base manifold for five dimensional supergravity solutions. In [10] regularity required the absence of closed time-like curves and this effectively excluded physical solutions whereas for our computations the non-trivial regularity conditions are essentially just positivity of the warp factor and we will find regular solutions.

The metric is

$$ds_{AH}^2 = \frac{1}{4}a_1^2a_2^2a_3^2d\eta^2 + \frac{1}{4}a_1^2\sigma_1^2 + \frac{1}{4}a_2^2\sigma_2^2 + \frac{1}{4}a_3^2\sigma_3^2 \tag{4.1}$$

with the SU(2) invariant one-forms satisfying $d\sigma_i = \frac{1}{2}\epsilon_{ijk}\sigma_j \wedge \sigma_k$ given by

$$\begin{aligned} \sigma_1 &= \cos\psi d\theta + \sin\psi \sin\theta d\phi \\ \sigma_2 &= \sin\psi d\theta - \cos\psi \sin\theta d\phi \\ \sigma_3 &= d\psi + \cos\theta d\phi \end{aligned}$$

and the a_i are subject to the following system of ODE's:

$$\frac{\dot{a}_1}{a_1} = \frac{1}{2} [(a_2 - a_3)^2 - a_1^2] \tag{4.2}$$

and cyclic permutations (dot is the derivative with respect to η). One defines new functions quadratic in the a_i :

$$w_1 = a_2a_3, \quad w_2 = a_1a_3, \quad w_3 = a_1a_2 \tag{4.3}$$

and then the system of ODE's is then

$$(w_1 + w_2)' = -2 \frac{w_1 w_2}{u^2} \tag{4.4}$$

$$(w_2 + w_3)' = -2 \frac{w_2 w_3}{u^2} \tag{4.5}$$

$$(w_3 + w_1)' = -2 \frac{w_3 w_1}{u^2} \tag{4.6}$$

(prime is derivative with respect to θ) with solution

$$w_1 = -uu' - \frac{1}{2}u^2 \csc \theta \tag{4.7}$$

$$w_2 = -uu' + \frac{1}{2}u^2 \cot \theta \tag{4.8}$$

$$w_3 = -uu' + \frac{1}{2}u^2 \csc \theta, \tag{4.9}$$

where

$$u = \frac{1}{\pi} \sqrt{\sin \theta} K \left(\sin^2 \frac{\theta}{2} \right) \tag{4.10}$$

and η is given in terms of θ through

$$u^2 d\eta = d\theta, \quad \eta = - \int_{\theta}^{\pi} \frac{d\theta}{u^2}. \tag{4.11}$$

For our gauge field ansatz we need an anti-self dual two form on M_{AH} , this is then guaranteed to solve Hermitian Yang-Mills without the need to construct the explicit Hyper-Kähler structure.⁵ In a classic paper on dualities [8], Sen gave an integral expression for exactly such an anti-self dual, harmonic two-form on M_{AH} but the appearance of this two-form dates back to the works [25–27]. Interestingly, from the work [10] we have the closed-form expression of this two-form

$$\Omega = h (a_1^2 dr \wedge \sigma_1 - \sigma_2 \wedge \sigma_3), \tag{4.12}$$

$$h = \frac{u^2}{w_1 \sin \frac{\theta}{2}}. \tag{4.13}$$

In [10] they consider self-dual forms but with a small modification of the frames this is made anti-self dual. More precisely our choice of frames is

$$e_0 = \frac{a_1 a_2 a_3}{2} d\eta, \quad e_i = \frac{a_i}{2} \sigma_i, \tag{4.14}$$

whereas in [10] an additional minus sign in e_0 was used. So we have locally

$$\Omega = -d(h \sigma_1). \tag{4.15}$$

In fact one can construct a triplet of anti-self dual forms Ω_- and a triplet of self-dual forms Ω_+ in a similar manner:

$$\Omega_{i-} = -d(h_i \sigma_i), \quad \Omega_{i+} = d(h_i^{-1} \sigma_i) \tag{4.16}$$

with

$$h_1 = \frac{u^2}{w_1 \sin \frac{\theta}{2}}, \quad h_2 = \frac{u^2}{w_2}, \quad h_3 = \frac{u^2}{w_3 \cos \frac{\theta}{2}}, \tag{4.17}$$

however only $\Omega_{1-} = \Omega$ is normalizable. Given that there is a single non-trivial two-cycle in M_{AH} one might be pleased to know that this normalizable form is dual to this two-cycle but there was no guarantee that the dual two-form would have an $SU(2)$ invariant representative.

⁵One could in principle write down the Hyper-Kähler structure using the results of [24] or by computing the Killing spinors.

4.1 Bianchi identity

We take our gauge field to be

$$F = \Omega \mathbf{q} \cdot \mathcal{H} \tag{4.18}$$

where $\mathcal{H} \in U(1)^{16}$ is in the Cartan subalgebra of $E_8 \times E_8$ or $SO(32)$ and \mathbf{q} the corresponding charge vector. The three-form flux is

$$\mathcal{H}_{(3)} = -\frac{H'}{4} \sigma^1 \wedge \sigma^2 \wedge \sigma^3 \tag{4.19}$$

and the Bianchi identity is

$$d *_4 dH = -2q^2 \Omega \wedge \Omega, \tag{4.20}$$

where $q^2 = \mathbf{q} \cdot \mathbf{q}$.

Quite remarkably, one can integrate this Poisson equation explicitly

$$H = h_0 + h_1 \eta + \frac{2q^2}{w_1} \tag{4.21}$$

where $\{h_0, h_1\}$ are constant coefficients of the s-wave harmonic functions on M_{AH} . The last term is manifestly negative definite for the whole region $0 \leq \theta \leq \pi$ but we will see that one can compensate for this by a choice of harmonic function and obtain a positive definite warpfactor.

4.2 Regularity

The regularity of M_{AH} has been previously studied in detail, we repeat it here to help determine regularity of our warp factor.

In the region $\theta \sim \pi$, we define a radial co-ordinate $r = -\log \cos \frac{\theta}{2}$ and using

$$K = r + \log(4) + \dots \tag{4.22}$$

$$u = \frac{\sqrt{2}}{\pi} r e^{-r/2} + \dots \tag{4.23}$$

$$w_1 = -\frac{r}{\pi^2} \tag{4.24}$$

we find that the metric is

$$ds_{AH}^2 = dr^2 + r^2(\sigma_1^2 + \sigma_2^2) + \sigma_3^2 + \dots \tag{4.25}$$

and

$$\frac{1}{w_1} = -\frac{\pi^2}{r} + \mathcal{O}(r^{-2}) \tag{4.26}$$

$$\eta = -\frac{\pi^2}{r} + \frac{\pi \log(4)}{r^2} + \mathcal{O}(r^{-3}) \tag{4.27}$$

so that the asymptotic expansion of the warp factor is

$$H = h_0 - \frac{h_1 + 2q^2}{r} + \dots \tag{4.28}$$

In the region $\theta \sim 0$, we define a new radial variable $\rho = \frac{\theta^2}{64}$ and the metric is

$$ds_{AH}^2 = d\rho^2 + 4\rho^2\sigma_1^2 + \frac{1}{16}(\sigma_2^2 + \sigma_3^2) + \dots \quad (4.29)$$

with

$$\frac{1}{w_1} = -4 + 32\rho^2 + \dots \quad (4.30)$$

$$\eta = \log \rho^2 + \dots \quad (4.31)$$

so that the IR expansion of the warp factor is

$$H = (h_0 - 8q^2) + 64q^2\rho^2 + \dots \quad (4.32)$$

From these expansions we see that with

$$h_0 > 8q^2, \quad h_1 = 0 \quad (4.33)$$

we have a positive warp factor which is regular everywhere. We define a rescaled radial coordinate near $\theta \sim \pi$ to be $\hat{r} = h_0^{1/2}r$ and $\hat{\rho} = h_0^{1/2}\rho$ near $\theta = 0$ so that

$$\theta \sim \pi: \quad ds_{10} = ds_{1,5}^2 + d\hat{r}^2 + \hat{r}^2(\sigma_1^2 + \sigma_2^2) + h_0\sigma_3^2 + \dots \quad (4.34)$$

$$\theta \sim 0: \quad ds_{10}^2 = ds_{1,5}^2 + d\hat{\rho}^2 + 4\hat{\rho}^2\sigma_1^2 + \frac{h_0}{16}(\sigma_2^2 + \sigma_3^2) + \dots \quad (4.35)$$

and see that the cost of a positive warp factor is that both the circle at infinity and the two-sphere at the bolt are large.

It is also important that the $\text{Tr}R_- \wedge R_-$ term in the Bianchi identity remains small compared to $\text{Tr}F \wedge F$. From explicit computations we find that the only possible divergences in $\text{Tr}R_- \wedge R_-$ appear through the warp factor as⁶ $\{H'/H, H''/H\}$ which by tuning h_0 can be made sufficiently small with respect to $\text{Tr}F \wedge F$. This confirms that our large charge approximation remains valid and these warped Atiyah-Hitchin solutions are good Heterotic backgrounds at leading order.

Alternatively we can obtain a positive warp factor through⁷

$$h_0 = 1, \quad h_1 < -2q^2. \quad (4.36)$$

This corresponds to smearing neutral five-branes on the S^2 at $\theta = 0$. Note that due to this smearing, at the IR ($\theta = 0$) the harmonic function parameterized by h_1 scales like a source in \mathbb{R}^2 . In the UV ($\theta = \pi$), due to the finite circle, the harmonic function scales like $\frac{1}{r}$ which is that of a source in \mathbb{R}^3 not \mathbb{R}^4 . The solution is of course singular for the usual reason that smeared branes are singular but this is of a good type and is resolved in string theory.

⁶An explicit computation using the Chern connection can be found in [14] and agrees with our conclusion here.

⁷The value of h_0 could be chosen to be another non-zero number $\mathcal{O}(q^0)$.

4.3 Five-brane charge

Computing the five-brane charge requires understanding some global features of M_{AH} . From (4.25) and (4.29) we see that there are two inequivalent, emergent $U(1)$ symmetries in the UV and IR, which are broken in the bulk. From [23] we know that a regular manifold requires the periodicities to be

$$0 \leq \psi \leq 2\pi, \quad 0 \leq \theta \leq \pi, \quad 0 \leq \phi \leq 2\pi \tag{4.37}$$

as well as that the free \mathbb{Z}_2 symmetry

$$I_1: \quad \theta \rightarrow \pi - \theta, \quad \phi \rightarrow \pi + \phi, \quad \psi \rightarrow -\psi \tag{4.38}$$

is enforced. The horizontal space in the UV is thus \mathbb{RP}^3/I_1 .

Using (4.28) we have

$$\mathcal{H}_{(3)} = [(h_1 + 2q^2) + \dots] \wedge \sigma^1 \wedge \sigma^2 \wedge \sigma^3 \tag{4.39}$$

compute the Maxwell five-brane charge to be

$$\mathcal{Q}_M = \frac{1}{4\pi\alpha'} \int_{\mathbb{RP}^3/I_1} \mathcal{H}_{(3)} = h_1 + 2q^2. \tag{4.40}$$

This is not required to be quantized. The Page charge is defined as in (3.17) and we find

$$\mathcal{Q}_P = h_1 \tag{4.41}$$

which must be integral.

4.4 Gauge field charge

The gauge field charge is computed using (4.12) and (4.18) and the IR expansion

$$h = -2 + \dots \tag{4.42}$$

Under the symmetry (4.38), the bolt remains a two sphere⁸ whose volume is 4π . We find

$$\begin{aligned} \frac{1}{2\pi} \int_{S^2} F &= 2\mathbf{q} \cdot \mathcal{H} \frac{1}{2\pi} \int_{S^2} \sigma_2 \wedge \sigma_3 \\ &= 4\mathbf{q} \cdot \mathcal{H} \in \mathbb{Z}. \end{aligned} \tag{4.43}$$

5 Fundamental string sources and AdS_3 solutions

Heterotic backgrounds with an $\mathbb{R}^{1,1}$ factor allow for the inclusion of F1-strings along $\mathbb{R}^{1,1}$ in addition to the magnetic five-branes. The electric source of three form flux induces a non-trivial warp factor and allows for AdS_3 solutions. To include these fundamental strings,

⁸As explained in [23] there is an additional, optional \mathbb{Z}_2 symmetry usually denoted I_3 which would convert the bolt into an \mathbb{RP}^3 .

we first consider internal eight-manifolds X_8 and then specialize the internal manifold to be a product of Hyper-Kähler manifolds.

The metric and three form are

$$ds_{10}^2 = e^{2A} ds_{1,1}^2 + ds_8^2 \tag{5.1}$$

$$H^{(3)} = \text{vol}_2 \wedge h^{(1)} + h^{(3)} \tag{5.2}$$

where $\text{vol}_2 = e^{2A} dx^0 \wedge dx^1$. Then we find that the BPS equations are a slight embellishment of those found in [28]:

$$\Psi \lrcorner d\Psi = d(\phi - A) \tag{5.3}$$

$$H_{(3)} = h_{(1)} \wedge \text{vol}_2 + h_{(3)} \tag{5.4}$$

$$h_{(1)} = -2dA \tag{5.5}$$

$$h_{(3)} = *_8 e^{2(\phi-A)} d(e^{2(A-\phi)} \Psi) \tag{5.6}$$

where Ψ is the Spin(8) structure on M_8 :⁹

$$\begin{aligned} \Psi = & e^{1234} + e^{1256} + e^{1278} + e^{3456} + e^{3478} + e^{5678} \\ & + e^{1357} - e^{1368} - e^{1458} - e^{1467} - e^{2358} - e^{2367} - e^{2457} + e^{2468} . \end{aligned} \tag{5.8}$$

We must supplement the BPS equations with the Bianchi identity (2.3) and then due to the non-trivial warp-factor A , one must also impose the three form flux equation of motion:

$$0 = d(e^{-2\phi} *_{10} H_{(3)}) \quad \Rightarrow \quad \begin{cases} 0 = d(e^{-2\phi} *_8 h_{(1)}) \\ 0 = d(e^{2(A-\phi)} *_8 h_{(3)}) \end{cases} . \tag{5.9}$$

5.1 Product of Hyper-Kähler manifolds

Our solutions with string and five-brane charges have a natural splitting of the internal eight manifold into a product of Hyper-Kähler manifolds¹⁰

$$ds_{10}^2 = e^{2A} ds_{\mathbb{R}^{1,1}}^2 + ds_{M_1}^2 + e^{2B} ds_{M_2}^2 \tag{5.10}$$

where M_i are both HyperKähler four manifolds, whose triplet of Kähler forms we denote

$$\{J_i, \text{Re } \Omega_i, \text{Im } \Omega_i\} . \tag{5.11}$$

The functions A, B depend only on the co-ordinates y_i of M_2 . The Spin(8) structure is given by

$$\Psi = \frac{1}{2} (J_1 \wedge J_1 + 2e^{2B} J_1 \wedge J_2 + e^{4B} J_2 \wedge J_2 + e^{2B} (\Omega_1 \wedge \Omega_2 + \bar{\Omega}_1 \wedge \bar{\Omega}_2)) . \tag{5.12}$$

⁹We note that with canonical holomorphic frames $E_i = e_{2i-1} + ie_{2i}$ such that $ds_8^2 = E_i \otimes \bar{E}_i$ the SU(4) structure is $J = \frac{1}{2} E_i \wedge \bar{E}_i$, $\Omega = E_1 \wedge E_2 \wedge E_3 \wedge E_4$ and

$$\Psi = \frac{1}{2} (J \wedge J + \Omega + \bar{\Omega}) . \tag{5.7}$$

¹⁰One might consider an additional warp factor in front of $ds_{M_1}^2$ however from [29, 30] we know that this must be constant.

We find the BPS conditions, Bianchi identity and equations of motion give¹¹

$$\phi = A + B \tag{5.13}$$

$$h_{(3)} = - *_{M_2} d e^{2B} \tag{5.14}$$

$$d *_{M_2} d e^{2B} = -\frac{1}{2} \alpha' \text{Tr} F \wedge F \tag{5.15}$$

$$0 = d *_{M_2} d e^{-2A} \tag{5.16}$$

$$0 = J_2 \lrcorner F \tag{5.17}$$

so we see that the only additional pieces of data from the equations in section 2 is that e^{2A} is harmonic on M_2 and the dilaton receives a shift proportional to A . For the Atiyah-Hitchin manifold we can smear F1-strings on the S^2 bolt in much the same way as we have described for smearing 5-branes on the bolt around (4.36), that is by

$$e^{2A} \sim \eta. \tag{5.18}$$

We will now be somewhat more explicit for the Gibbons-Hawking spaces.

5.2 AdS₃ from Gibbons-Hawking

When M_2 is a Gibbons-Hawking space, the U(1) invariant harmonic functions are

$$e^{-2A} = 1 + \sum_r \frac{\hat{q}_r}{|\mathbf{x} - \mathbf{x}_r|} \tag{5.19}$$

corresponding to strings placed along $\mathbb{R}^{1,1}$ and at fixed points of ∂_τ on M_2 .

If in addition we choose to place these strings at poles of V we recover AdS₃ geometries near such a pole. We put k' poles of V as well as the strings at $\mathbf{x}_r = \mathbf{x}_i = 0$ then in the vicinity of x_i we have

$$e^{2A} = \frac{r}{\hat{q}_0}, \dots \quad e^{2B} = \frac{1}{r} \frac{\alpha'}{4m} Q_5 + \dots, \quad V = \frac{2mk'}{r} + \dots \tag{5.20}$$

$$\tag{5.21}$$

so that

$$ds_{10}^2 = \frac{r}{\hat{q}_0} ds_{1,1}^2 + ds_{M_1}^2 + 2\alpha' k'^2 Q_5 \left[\frac{1}{4} \frac{dr^2}{r^2} + ds_{S^3/\mathbb{Z}_k}^2 \right] \tag{5.22}$$

$$= 2\alpha' k'^2 Q_5 \left[ds_{AdS_3}^2 + ds_{S^3/\mathbb{Z}_k}^2 \right] + ds_{M_1}^2 \tag{5.23}$$

$$e^{2\phi} = \frac{\alpha' Q_5}{4m\hat{q}_0} \tag{5.24}$$

where $r = \rho^2$. The F1-charge is given as usual by

$$Q_1 = \frac{4m\hat{q}_0 \text{vol}(M_1)}{\alpha'^3}. \tag{5.25}$$

The gauge field vanishes in this limit and the background is sourced by three-form flux.

¹¹Note that is $E = e^B \tilde{E}$ is an 8d frame $*_8 E \wedge J_1 \wedge J_1 = 2 *_8 E \wedge \text{vol}_{M_1} = 2e^{3B} *__{M_2} \tilde{E} = 2e^{2B} *__{M_2} E$.

6 Conclusions

The key aspect of our solutions with Abelian gauge bundles is that we have taken a large charge limit and consistently suppressed the $\text{Tr } R \wedge R$ term in the Bianchi identity, which is subdominant at leading order in the expansion in $\frac{1}{q^2}$. We have shown how this large charge limit can lead to exact supersymmetric flux backgrounds, and it is particularly interesting the Atiyah-Hitchin manifold can provide a regular background. This configuration requires some ingenuity to counteract the negative mass and result in a background of the correct signature. This Atiyah-Hitchin based solution is distinctly different from those based on Gibbons-Hawking; while the latter can be viewed as marginal deformations of the orbifold of the CHS solutions the finite two-cycle in the Atiyah-Hitchin manifold cannot be blown down. As such we do not have a worldsheet theory from which we can imagine obtaining this as the background geometry.

In these backgrounds, the gauge fields are completely solved for by using the Hermitian Yang-Mills equations which then provide a source for the three-form flux. It is conceivable that non-Abelian bundles could be constructed such that $\text{Tr } F \wedge F$ dominates $\text{Tr } R \wedge R$ everywhere.¹² Since the Kronheimer-Nakajima construction [33] gives a solution of all instantons on ALE Gibbons-Hawking spaces, one could possibly even construct such instantons, however most instantons will provide a source the Bianchi identity whose solution is a general function of four variables and thus unsolvable. A particularly neat class of instantons is based on the 't Hooft ansatz [34]:

$$\begin{aligned} A_0 &= \frac{1}{2} \vec{G} \cdot \vec{\sigma}, & \vec{A} &= \frac{1}{2} \left[\vec{\omega}(\vec{G} \cdot \vec{\sigma}) - V(\vec{G} \times \vec{\sigma}) \right] \\ \vec{G} &= -V^{-1} \vec{\nabla} \log f \end{aligned} \tag{6.1}$$

with f harmonic on \mathbb{R}^3 . For finite action, the centers of f are constrained to lie at the poles of V . These instantons can have large $\text{Tr } F \wedge F$ in the limit of large number of poles of f but $\text{Tr } R \wedge R$ will not be suppressed.

There are numerous directions for progress on the worldsheet description of these backgrounds. The elliptic genus for type II on ALE spaces has been computed recently [35] based on general developments in this field [36] and we expect to be able to provide a similar solution for these Heterotic models or the type II flux backgrounds of section 2.1. It would also be interesting to provide an exactly solvable worldsheet model of the near-horizon region of the multi-center Gibbons-Hawking backgrounds, generalizing the gauged WZW model of the two-centered solution.

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¹²Interesting five dimensional solutions with non-Abelian gauge fields have appeared recently [31] and the lift to the Heterotic string has been discussed [32]. However it is not clear to us how these solutions will solve the exact Bianchi identity.

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A Eguchi-Hanson

When $k = 2$ and $\epsilon = 0$, the explicit co-ordinate transformation is known [37] from the Gibbons-Hawking space to the Eguchi-Hanson space [38]. In Cartesian co-ordinates the two center ALE Gibbons-Hawking space has

$$\omega = \left[\frac{z - a^2/8}{\sqrt{x^2 + y^2 + (z - a^2/8)^2}} + \frac{z + a^2/8}{\sqrt{x^2 + y^2 + (z + a^2/8)^2}} \right] d\left(\tan^{-1} \frac{y}{x}\right) \quad (\text{A.1})$$

$$V = \frac{1}{\sqrt{x^2 + y^2 + (z - a^2/8)^2}} + \frac{1}{\sqrt{x^2 + y^2 + (z + a^2/8)^2}}. \quad (\text{A.2})$$

Following [37] we have ($a \leq r$):

$$\begin{aligned} x &= \frac{a^2}{8} \sqrt{\frac{r^4}{a^4} - 1} \sin \theta \cos \psi \\ y &= \frac{a^2}{8} \sqrt{\frac{r^4}{a^4} - 1} \sin \theta \sin \psi \\ z &= \frac{1}{8} r^2 \cos \theta \end{aligned}$$

so that

$$\begin{aligned} V &= \frac{16}{a^2} \frac{\frac{r^2}{a^2}}{\frac{r^4}{a^4} - \cos^2 \theta}, \\ \omega &= \frac{2 \cos \theta (\frac{r^2}{a^2} - 1)}{\frac{r^2}{a^2} - \cos \theta} d\psi. \end{aligned} \quad (\text{A.3})$$

As an example, we write explicitly the solution for Heterotic five-branes on Eguchi-Hanson with additional F1-strings.¹³

$$ds_{M_2}^2 = \frac{dr^2}{f^2} + \frac{r^2}{4} [\sigma_1^2 + \sigma_2^2 + f^2 \sigma_3^2] \quad (\text{A.4})$$

$$f^2 = 1 - \frac{a^4}{r^4} \quad (\text{A.5})$$

$$h_{(3)} = 2f^2 r^3 (e^{2B})' \sigma_1 \wedge \sigma_2 \wedge \sigma_3$$

$$F = d\left(\frac{a^2}{r^2} \eta\right) \quad (\text{A.6})$$

$$e^{2B} = 1 + \frac{8\alpha' q^2}{r^2} + \frac{Q_5}{8a^2} \log \left[\frac{r^2/a^2 - 1}{r^2/a^2 + 1} \right] \quad (\text{A.7})$$

¹³One can take M_1 to be T^4 or $K3$ with the Ricci-flat metric.

$$e^{-2A} = 1 + \frac{Q_1}{a^2} \log \left[\frac{r^2/a^2 - 1}{r^2/a^2 + 1} \right] \tag{A.8}$$

$$e^{2\Phi} = e^{2(A+B)} \tag{A.9}$$

In addition to the Heterotic five-branes which resolve the singularity, there are Q_1 mobile F1-strings and Q_5 NS5-branes smeared on the blown-up S^2 . Due the smearing of the strings, the near horizon limit has a log-singularity at $r \sim a$ in the warp factor e^{2A} and thus there is no enhancement to AdS_3 . In the blow-down limit $a \rightarrow 0$ where the Eguchi-Hanson space becomes $\mathbb{C}^2/\mathbb{Z}_2$, the gauge field vanishes and we get the \mathbb{Z}_2 orbifold of the usual F1-NS5-solution, the near-horizon limit is $\text{AdS}_3 \times S^3/\mathbb{Z}_2 \times M_1$.

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